Radiation and laminar forced convection of non-Newtonian fluid in a circular tube

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Interaction of radiation with thermally developing laminar forced convection of power-law, non-Newtonian, absorbing, emitting, isotropically scattering, gray fluid through an isothermal circular tube with a black boundary is investigated. The energy equation is solved by an implicit finite-difference scheme, while the radiation part of the problem is solved by the collocation method. Results are presented for the effects of conduction-to-radiation parameter, single scattering albedo, optical thickness and the inlet temperature on the local Nusselt number along the tube for the case of power-law index $n=1/3$, 1 and 3, where the case $n=1$ corresponds to the Newtonian fluid.

Keywords: radiation; forced convection; non-Newtonian flow; internal flow; laminar flow

Introduction

Heat transfer by simultaneous radiation and convection is important in many engineering applications such as those involving furnaces, combustion chambers and high temperature heat exchangers. Some studies[1–4] have been reported on the interaction of radiation without scattering effects in circular ducts. Azad and Modest[5] investigated the problem of combined radiation and turbulent forced convection in absorbing, emitting and linearly anisotropic scattering gas–particulate flow through a circular tube. Yener and Fong[6] studied the interaction of radiation and laminar forced convection in a circular pipe by treating the radiation part rigorously, but neglecting the emission of the fluid.

Furthermore, all of the foregoing papers dealt with the problems of combined radiation and convection for Newtonian fluids through ducts. The assumption of Newtonian fluids is true for all gases and ordinary fluids such as water and oils. However, some special fluids such as colloids, emulsions and slurries exhibit the behavior of non-Newtonian fluids, i.e., nonlinear relationship between shear stress and velocity gradient. In this work, the interaction of radiation and convection for a thermally developing, hydrodynamically fully developed, steady laminar flow of non-Newtonian fluid in a circular duct has been investigated.

Analysis

Consider thermally-developing hydrodynamically-developed steady laminar flow of an absorbing, emitting, isotropically scattering gray non-Newtonian fluid through a circular duct with an isothermal black wall. The fluid at a uniform temperature T_i enters the heated section of the tube at the origin of the coordinate $x=0$ with a fully developed velocity profile $u(r)$, while the tube wall is kept at a constant temperature T_w for $x>0$, as shown in Figure 1.

Assuming constant thermo-physical properties for the fluid, neglecting viscous dissipation, axial conduction and axial radiation for $\mathrm{Re}\,\mathrm{Pr}\gg 1$,[4] the energy equation in dimensionless form can be written as

$$\frac{U(\eta)}{4}\frac{\partial\Theta(\eta,\xi)}{\partial\xi}=\frac{\partial^2\Theta(\eta,\xi)}{\partial\eta^2}+\frac{1}{\eta}\frac{\partial\Theta(\eta,\xi)}{\partial\eta}-\frac{1}{\eta N}\frac{\partial[\eta Q^r(\eta,\xi)]}{\partial\eta}\qquad 0<\eta<1,\quad \xi>0 \tag{1}$$

subject to the boundary and inlet conditions

$$\frac{\partial\Theta(0,\xi)}{\partial\eta}=0\qquad \eta=0,\quad \xi>0 \tag{2}$$

$$\Theta(1,\xi)=\Theta_w=1\qquad \eta=1,\quad \xi>0 \tag{3}$$

$$\Theta(\eta,0)=\Theta_i\qquad 0\le\eta\le 1,\quad \xi=0 \tag{4}$$

where $\Theta_i=T_i/T_w$ and the normalized velocity profile for power-law non-Newtonian fluid is taken as[7]

$$U(\eta)=\frac{1+3n}{1+n}\left(1-\eta^{(1+n)/n}\right) \tag{5}$$

Here the power-law index $0\le n<1$ characterizes a dilatant fluid, $n=1$ a Newtonian fluid and $1<n<\infty$ a pseudoplastic fluid.

For the radiation part of the problem, we assume an absorbing, emitting, isotropically scattering, gray fluid in a circular tube with black wall maintained at a constant temperature T_w. The equation of radiation transfer and the boundary condition are given by[8]

$$\left[\sin\psi\cos\phi\frac{\partial}{\partial\tau}-\frac{1}{\tau}\sin\psi\sin\phi\frac{\partial}{\partial\phi}+1\right]I(\tau,\psi,\phi)=S(\tau)+\frac{\omega}{4\pi}G(\tau)\qquad 0<\tau<\tau_o,\quad 0\le\psi\le\pi,\quad 0\le\phi\le 2\pi \tag{6}$$

and

$$I(\tau_o,\psi,\phi)=\bar{n}^2\bar{\sigma}T_w^4/\pi\qquad 0\le\psi\le\pi,\quad 0\le\phi\le 2\pi \tag{7}$$

where $I(\tau,\psi,\phi)$ is the radiation intensity, τ is the optical variable, ψ and ϕ are the polar and azimuthal angles, respectively, ω is the single scattering albedo, and $G(\tau)$ is the incident radiation defined as

$$G(\tau)=\int_{\phi=0}^{2\pi}\int_{\psi=0}^{\pi}I(\tau,\psi,\phi)\sin\psi\,\mathrm{d}\psi\,\mathrm{d}\phi \tag{8}$$

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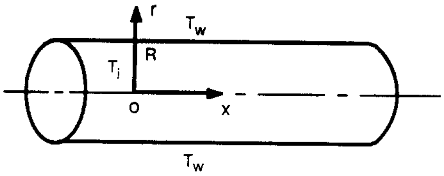

Figure 1 Geometry and coordinates

and $S(\tau)$ is the source term, given by

$$S(\tau)=(1-\omega)\bar{n}^2\bar{\sigma}T^4(\tau)/\pi \tag{9}$$

The radiation problem defined above is transformed into Fredholm integral equation for the incident radiation $G(\tau)$ by following an approach described in Ref. 9,

$$G^*(\tau)=Y(\tau)+\int_{t=0}^{\tau_0}[S^*(t)+\omega G^*(t)]L_0(\tau,t)t\,dt \tag{10}$$

where

$$G^*(\tau)=G(\tau)/4\bar{n}^2\bar{\sigma}T_w^4 \tag{11a}$$

$$Y(\tau)=\frac{2}{\pi}\int_{\phi=0}^{\pi/2}\int_{\psi=0}^{\pi/2}\cosh\left[\frac{\tau\cos\phi}{\sin\psi}\right]\times\exp\left[-\frac{\sqrt{\tau_0^2-\tau^2\sin^2\phi}}{\sin\psi}\right]\sin\psi\,d\psi\,d\phi \tag{11b}$$

$$S^*(\tau)=(1-\omega)\Theta^4(\tau) \tag{11c}$$

and the kernel $L_0(\tau,t)$ is defined by

$$L_0(\tau,t)=\begin{cases}\displaystyle\int_{\mu=0}^{1}\frac{K_0(\tau/\mu)I_0(t/\mu)}{\mu^2}\,d\mu & \tau>t\\ \displaystyle(-1)^n\int_{\mu=0}^{1}\frac{I_0(\tau/\mu)K_0(t/\mu)}{\mu^2}\,d\mu & \tau<t\end{cases} \tag{11d}$$

Here I_0 and K_0 are the modified Bessel functions.

We assume the dimensionless source term $S^*(\tau)$ can be expressed as a polynomial of order M in the even power of τ because of symmetry in a solid cylinder, i.e.,

$$S^*(\tau)=(1-\omega)\sum_{m=0}^{M}s_m\tau^m \qquad m=0,2,4,\ldots,M \tag{12}$$

To solve the integral equation 10 with the collocation method, the dimensionless incident radiation $G^*(\tau)$ is expanded as

$$G^*(\tau)=\sum_{j=1}^{J}a_j\frac{I_0(\tau/\lambda_j)}{I_0(\tau_0/\lambda_j)} \tag{13}$$

where $J=(L+1)/2$ with L being odd, a_j are the unknown expansion coefficients and $I_0(x)$ is the modified Bessel function of the first kind of order zero. The eigenvalues λ^2 are determined from the solution of the eigenvalue problem[10]

$$\frac{l(l-1)}{h_lh_{l-1}}g_{l-2}(\lambda)+\frac{1}{h_l}\left[\frac{(l+1)^2}{h_{l+1}}+\frac{l^2}{h_{l-1}}\right]g_l(\lambda)+\frac{(l+2)(l+1)}{h_{l+1}h_l}g_{l+2}(\lambda)=\lambda^2g_l(\lambda) \tag{14}$$

where $l=0,2,4,\ldots,L-1$ and g_l are Chandrasekhar's polynomial defined by[11]

$$(l+1)g_{l+1}(\lambda)=h_l\lambda g_l(\lambda)-lg_{l-1}(\lambda) \tag{15}$$

The coefficients h_l are determined from

$$h_l=2l+1-\omega\delta_{0,l} \tag{16}$$

where $\delta_{i,j}$ denotes the Kronecker delta. Note that Equation 14 is valid only for $\omega<1$; and some modifications are required for the special case of $\omega=1$.[12] The eigenvalues λ^2 are determined by solving Equation 14 with a FORTRAN program given in the EISPACK program package.[13]

To apply the collocation method, the representation given by Equation 13 is introduced into Equation 10 and a set of collocation points τ_c, $c=1,2,\ldots,J$ are chosen. The following system of linear algebraic equations result for the unknown

Notation

a_j	Expansion coefficients defined by Equation 13
D_h	$=2R$, hydraulic diameter
$G(\tau,\xi)$	Incident radiation
$G^*(\tau,\xi)$	$=G/4\bar{n}^2\bar{\sigma}T_w^4$, dimensionless incident radiation
$I(\tau,\psi,\phi)$	Radiation intensity
$I_0(x)$	The first kind modified Bessel function of order zero
J	$=(L+1)/2$
k	Thermal conductivity
L	An odd number used in Equation 14
$L_0(\tau,t)$	Kernel defined by Equation 11d
M	An even number used in Equation 12
N	$=(k/R)/4\bar{n}^2\bar{\sigma}T_w^3$, conduction-to-radiation parameter
n	Power-law index for non-Newtonian fluid
$\bar{n}$	Refractive index
Pr	$=\nu/\alpha$, Prandtl number
$Q^r(\tau,\xi)$	$=q^r/4\bar{n}^2\bar{\sigma}T_w^4$, dimensionless radiation heat flux
$q^r(\tau,\xi)$	Radiation heat flux
$q_w^T(\xi)$	Total heat flux to the fluid at the wall
R	Radius of circular duct
Re	$=U_mD_h/\nu$, Reynolds number
r	Radial variable
$S(\tau,\xi)$	$=$source term
$S^*(\tau,\xi)$	$=(1-\omega)\Theta^4$, dimensionless source term
s_m	Expansion coefficients defined by Equation 12
$T(\eta,\xi)$	Temperature distribution
T_i	Inlet temperature
$T_m(\xi)$	Mean temperature
T_w	Wall temperature
$U(\eta)$	Normalized velocity profile
U_m	Mean velocity
x	Axial distance along the duct

Greek symbols

α	Thermal diffusivity
$\delta_{i,j}$	Kronecker delta
ε	Convergence criterion
ψ	Polar angle
ϕ	Azimuthal angle
η	$=r/R$, dimensionless radial variable
λ	Eigenvalue
ν	Kinematic viscosity
$\Theta(\eta,\xi)$	$=T/T_w$, dimensionless temperature distribution
Θ_i	$=T_i/T_w$, dimensionless inlet temperature
$\bar{\sigma}$	Stefan–Boltzmann constant
τ	Optical variable
τ_o	Optical thickness
τ_c	Collocation points
ω	Single scattering albedo
ξ	$=(x/D_h)/(\text{Re Pr})$, dimensionless axial coordinate

expansion coefficients a_j:

$$\sum_{j=1}^{J} \frac{a_j}{I_0(\tau_0/\lambda_j)}\left[I_0(\tau_c/\lambda_j)-\omega\int_{t=0}^{\tau_0} tL_0(\tau_c,t)I_0\left(\frac{t}{\lambda_j}\right)\mathrm{d}t\right]$$
$$=Y(\tau_c)+\int_{t=0}^{\tau_0} S^*(t)L_0(\tau_c,t)t\,\mathrm{d}t \qquad c=1,2,\ldots,J \tag{17a}$$

where the collocation points τ_c are the zeros of the Chebysheff polynomial[14] which are shifted to the interval $[0, \tau_o]$, i.e.,

$$\tau_c=\frac{\tau_o}{2}\left[1+\cos\left[\frac{2c-1}{2J}\pi\right]\right] \qquad c=1,2,\ldots,J \tag{17b}$$

The number of collocation points J used in the calculation varies from 15 for small optical thicknesses (i.e., $\tau_o \le 1$) to 20 for large optical thicknesses (i.e., $\tau_o > 1$).

For a given temperature distribution, the expansion coefficients a_j are determined from the solution of the system of Equations 17a,b and $G^*(\tau)$ is computed from Equation 13.

The divergence of radiation flux appearing in Equation 1 is related to $G^*(\tau)$ by[8]

$$\frac{1}{\eta}\frac{\partial[\eta Q^r(\eta,\xi)]}{\partial\eta}=\frac{\tau_o}{\tau}\frac{\partial[\tau Q^r(\tau,\xi)]}{\partial\tau}=\tau_o(1-\omega)[\Theta^4(\tau,\xi)-G^*(\tau,\xi)] \tag{18}$$

The case $\omega=1$ is not considered in the present problem, because for $\omega=1$ the divergence of the radiation flux vanishes as apparent from Equation 18, hence the radiation and convection problems are uncoupled.

Introducing the divergence of radiation flux given by Equation 18 into the energy equation, we obtain

$$\frac{U(\eta)}{4}\frac{\partial\Theta(\eta,\xi)}{\partial\xi}=\frac{\partial^2\Theta(\eta,\xi)}{\partial\eta^2}+\frac{1}{\eta}\frac{\partial\Theta(\eta,\xi)}{\partial\eta}-A[\Theta^4(\tau,\xi)-G^*(\tau,\xi)]$$
$$0<\eta<1, \quad \xi>0 \tag{19}$$

where $A=(1-\omega)\tau_0/N$. At the center of the cylinder, $\eta=0$, this equation should be replaced by

$$\frac{U(\eta)}{4}\frac{\partial\Theta(0,\xi)}{\partial\xi}=2\frac{\partial^2\Theta(0,\xi)}{\partial\eta^2}-A[\Theta^4(0,\xi)-G^*(0,\xi)]$$
$$\eta=0, \quad \xi>0 \tag{20}$$

Therefore, Equations 19 and 20 together with the boundary condition at $\eta=1$, (i.e., Equation 3) and the inlet condition, (i.e., Equation 4) represent a nonlinear boundary-value problem.

Because the radiation parts in Equations 19 and 20 depend on the temperature distribution $\Theta(\eta,\xi)$, iteration is needed at each $\Delta\xi$. That is, a temperature distribution, say, $\Theta^0(\eta,\xi)$, is first guessed, and the radiation part is solved by the collocation method and the incident radiation term $G^*(\tau)$ is determined. Knowing $G^*(\tau)$, the energy equation is then solved to obtain a new temperature $\Theta^1(\eta,\xi)$ by using an implicit finite difference scheme. Successive iteration is continued until $\Theta^m(\eta,\xi)$ converged to ε, i.e., $|\Theta^m(\eta,\xi)-\Theta^{m-1}(\eta,\xi)|<\varepsilon$ where ε is a small prescribed quantity. In the present calculations the value of ε is set to be 10^{-5}. Upon convergence, the solution is advanced to the next $\Delta\xi$. For the calculations, 41 mesh points were chosen in the radial direction η.

Once the temperature distribution $\Theta(\eta,\xi)$ and the incident radiation are available, the dimensionless radiation heat flux $Q^r(\tau,\xi)$ is computed by integrating Equation 18, to yield

$$Q^r(\tau,\xi)=\frac{1-\omega}{\tau}\int_0^{\tau} t[\Theta^4(t,\xi)-G^*(t,\xi)]\,\mathrm{d}t \tag{21}$$

The local Nusselt number is determined from its definition

$$\mathrm{Nu}(\xi)=\frac{q_w^T(\xi)}{k(T_m-T_w)/D_h} \tag{22a}$$

where the total heat flux at the wall $q_w^T(\xi)$ is obtained from

$$\frac{q_w^T(\xi)}{2kT_w/D_h}=-\frac{\partial\Theta(1,\xi)}{\partial\eta}+\frac{1}{N}Q^r(1,\xi) \tag{22b}$$

and the mean temperature $T_m(\xi)$ is given by

$$\frac{T_m(\xi)}{T_w}=\Theta_m(\xi)=\frac{2(1+3n)}{1+n}\int_0^1 \Theta(\eta,\xi)(1-\eta^{(1+n)/n})\eta\,\mathrm{d}\eta \tag{22c}$$

Finally, the expression for the local Nusselt number becomes

$$\mathrm{Nu}(\xi)=\frac{2}{\Theta_m(\xi)-1}\left[-\frac{\partial\Theta(1,\xi)}{\partial\eta}+\frac{1}{N}Q^r(1,\xi)\right] \tag{23}$$

which consists of a conduction and a radiation terms.

Results and discussion

Figures 2–5 show the effects of the conduction-to-radiation parameter N, single scattering albedo ω, optical thickness τ_o and inlet temperature Θ_i on the local Nusselt number Nu along the tube for each of the power-law index $n=1/3$, 1 and 3, where $n=1$ corresponds to the Newtonian fluid. In all these figures, the local Nusselt number decreases with increasing n because the flow velocity near the wall decreases as n increases; as a result, heat transfer from the wall is reduced. In each of these curves, when radiation is present, the local Nusselt number shows a minimum and with stronger radiation the minimum appears to be shifted toward smaller values of ξ. The reason for this minimum is the fact that the radiation contribution to the local Nusselt number increases monotonically with increasing ξ whereas the conduction contribution to the local Nusselt number decreases monotonically with increasing ξ.

In Figure 2, the results are presented to show the effects of the conduction-to-radiation parameter for the cases $N=0.02$, 0.05 and 0.5 by setting $\omega=0.5$, $\tau_o=1$, and $\Theta_i=0.5$. The values of $N=0.5$ and $N=0.02$ correspond to weak and strong radiation,

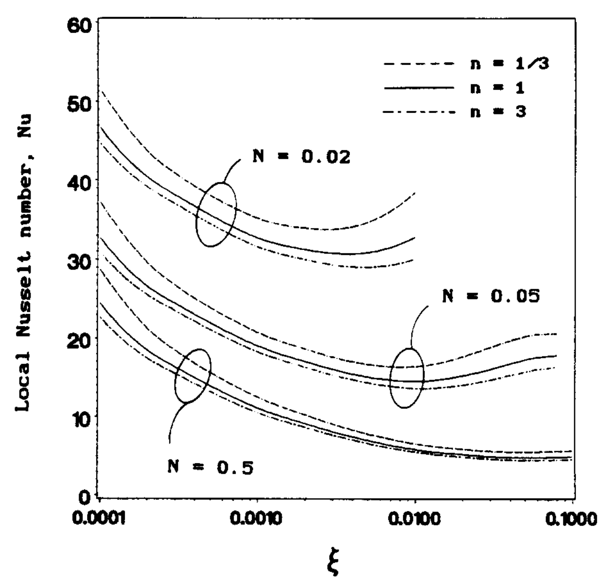

Figure 2 The effects of the conduction-to-radiation parameter N on the local Nusselt number for different fluids for $\omega=0.5$, $\tau_o=1$ and $\Theta_i=0.5$

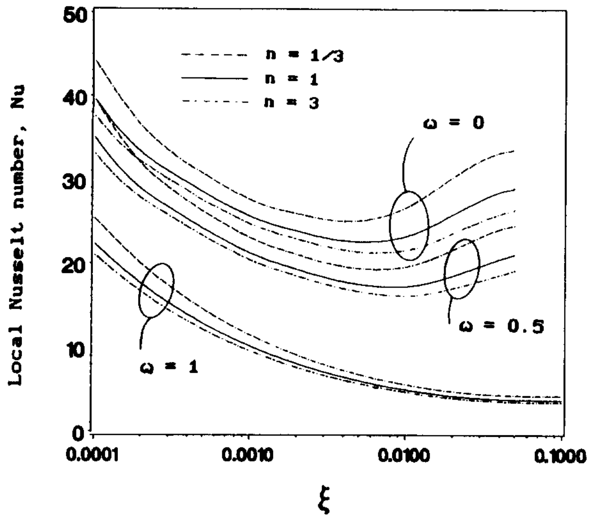

Figure 3 The effects of the single scattering albedo ω on the local Nusselt number for different fluids for $N=0.04$, $\tau_o=1$, and $\Theta_i=0.5$

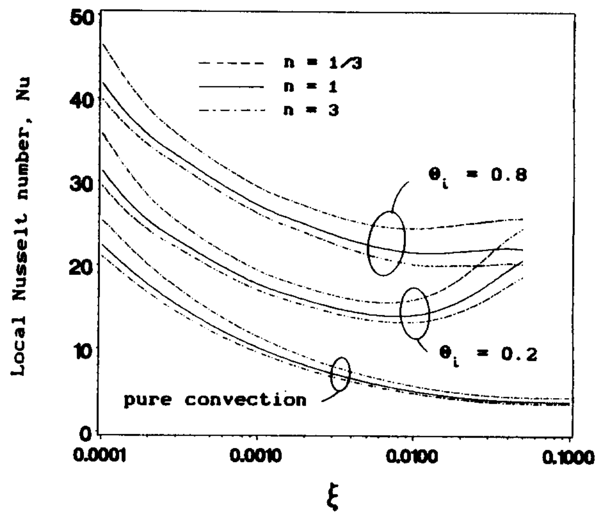

Figure 5 The effects of the inlet temperature Θ_i on the local Nusselt number for different fluids for $N=0.05$, $\omega=0.2$, and $\tau_o=1$

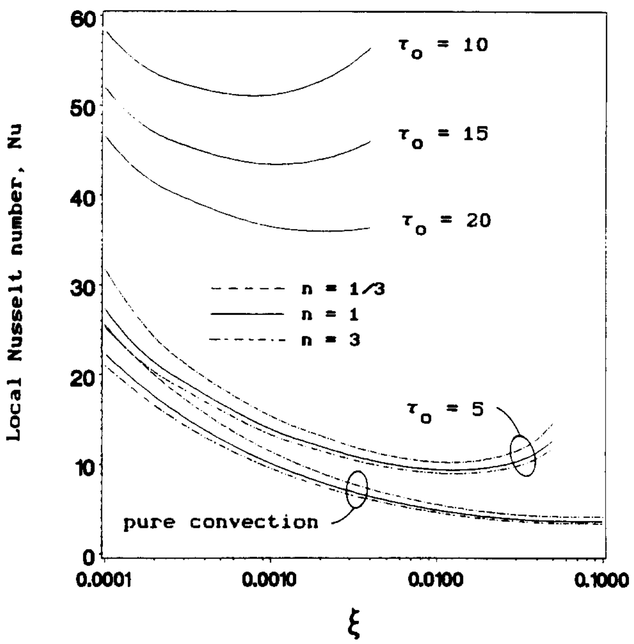

Figure 4 The effects of the optical thickness τ_o on the local Nusselt number for different fluids for $N=0.4$, $\omega=0$, and $\Theta_i=0.5$

respectively. The local Nusselt number increases with decreasing N, because stronger radiation helps to increase the heat transfer rate, hence the Nusselt number. The power-law index n has the significant effects on the local Nusselt number for small values of N.

Figure 3 shows the effects of single scattering albedo for the cases $\omega=0$, 0.5 and 1 by setting $N=0.04$, $\tau_o=1$ and $\Theta_i=0.5$. The values of single scattering albedo $\omega=0$ and $\omega=1$ corresponding, respectively, to nonscattering and pure scattering cases have the highest and the lowest local Nusselt number, respectively. The case $\omega=1$ is identical to pure convection. The effect of the power-law index n is the most significant on the local Nusselt number for $\omega=0$.

Figure 4 illustrates the effects of optical thickness τ_o on the local Nusselt number by setting $N=0.4$, $\omega=0$, and $\Theta_i=0.5$. The values of the optical thickness considered include $\tau_o=5$, 10, 15, and 20. As the radiative heat flux vanishes in both limits of $\tau_o=0$ and $\tau_o\to\infty$, we expect that the highest radiative heat flux occurs somewhere in the intermediate optical thickness. As a result, the local Nusselt number first increases with increasing optical thickness until some critical value of τ_o is reached and then, beyond that point the trend is reversed to attain an asymptotic value, i.e., the pure convection case. This is the reason why the curves for $\tau_o=15$ and 20 lie between the curves for $\tau_o=5$ and $\tau_o=10$.

Finally, Figure 5 is prepared to show the effects of the inlet temperature Θ_i on the local Nusselt number for the cases $\Theta_i=0.2$, and $\Theta_i=0.8$, by setting the other parameters as $N=0.05$, $\omega=0.2$ and $\tau_o=1$. With increasing Θ_i, the local Nusselt number increases because the mean temperature, hence, the radiative heat flux increases significantly. However, the effects of Θ_i diminish gradually at the locations far away from the inlet, because the temperatures for different values of Θ_i are indistinguishable and close to the wall temperature.

All the numerical calculations in this work were performed on the IBM 3081 computer. A few seconds of CPU time is required to solve the pure convection case. However, the cases of simultaneous convection and radiation require larger CPU times as a result of calculations of the radiation terms as well as the iteration scheme for the purpose of convergence. The value of the conduction-to-radiation parameter significantly affects the computing time. For example, in Figure 2, decreasing N from 0.5 to 0.02 increases the CPU time from about 30 sec to 2 min.

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